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Plenty of Nothing: Black Hole Entropy in Induced Gravity

V.P. Frolov and D.V. Fursaev

Department of Physics, University of Alberta, Edmonton, Canada T6G 2J1
e-mails: frolov, dfursaev@phys.ualberta.ca

Abstract

We demonstrate how Sakharov's idea of induced gravity allows one to explain the statistical-mechanical origin of the entropy of a black hole. According to this idea, gravity becomes dynamical as the result of quantum effects in the system of heavy constituents of the underlying theory. The black hole entropy is related to the properties of the vacuum in the induced gravity in the presence of the horizon. We obtain the Bekenstein-Hawking entropy by direct counting the states of the constituents.

There are physical phenomena that allow a simple description but require tremendous efforts to explain them. Without doubts the origin of black hole entropy is such a problem. A black hole of mass M radiates as a heated body [1] with temperature $T_H = (8\pi GM)^{-1}$ and has entropy [2]

$$S^{BH} = \frac{\mathcal{A}}{4G}\,, \tag{1}$$

where $\mathcal{A}$ is the surface area of the black hole ($\mathcal{A} = 16\pi G^2M^2$), G is Newton's constant, and $c = \hbar = 1$. Statistical mechanics relates entropy to the measure of disorder and qualitatively it is the logarithm of the number of microscopically different states available for given values of the macroscopical parameters. Are there internal degrees of freedom that are responsible for the Bekenstein-Hawking entropy S^{BH}? This is the question that physicists were trying to answer for almost 25 years.

What makes the problem of black hole entropy so intriguing? Before discussing more technical aspects let us make simple estimations. Consider for example a supermassive black hole of 10^9 solar mass. According to (1), its entropy is about 10^{95}. This is seven orders of magnitude larger than the entropy of the other matter in the visible part of the Universe. What makes things even more complicated, a black hole is simply an empty

space-time with a strong gravitational field and ... nothing more. Really, a black hole is "plenty of nothing", or if we put this in a more physical way, the phenomenon we are dealing with is a vacuum in the strong gravitational field. This conclusion leaves us practically no other choice but to try to relate the entropy of the black hole to properties of the physical vacuum in the strong gravitational field.

The black hole entropy is of the same order of magnitude as the logarithm of the number of different ways to distribute two signs $+$ and $-$ over the cells of Planckian size on the horizon surface. This estimation suggests that a reasonable microscopical explanation of the Bekenstein-Hawking entropy must be based on the quantum gravity, this Holy Grail of the theoretical physics. The superstring theory is the best what we have and what is often considered as the modern version of quantum gravity. Recent observation that the Bekenstein-Hawking entropy can be obtained by counting of string (and/or D-brane) states is a very interesting and important result.

Still there are questions. The string calculations essentially use supersymmetry and are mainly restricted to extreme and near-extreme black holes. Moreover each model requires new calculations. And the last but not the least it remains unclear why the entropy of a black hole is universal and does not depend on the details of the theory at Planckian scales. Note that the black hole thermodynamics follows from the low-energy gravitational theory. That is why one can expect that only a few fundamental properties of quantum gravity but not its concrete details are really important for the statistical-mechanical explanation of the black hole entropy.

In the string theory the low-energy gravity with finite Newton's constant arises as the collective phenomenon and is the result of quantum excitations of constituents (strings) of the underlying theory. There is certain similarity between this mechanism and Sakharov's induced gravity [3]. The low-energy effective action $W[g]$ in the induced gravity is defined as a quantum average of the constituent fields Φ propagating in a given external gravitational background g

$$\exp(-W[g]) = \int \mathcal{D}\Phi \exp(-I[\Phi, g]) \quad . \tag{2}$$

The Sakharov's basic assumption is that the gravity becomes dynamical only as the result of quantum effects of the constituent fields. The gravitons in this picture are analogous to the phonon field describing collective excitations of a crystal lattice in the low-temperature limit of the theory. Search for the statistical-mechanical origin of the black hole entropy in Sakharov's approach [4] might help to understand the universality of S^{BH}. Here we describe the mechanism of generation of the Bekenstein-Hawking entropy in the induced gravity [5, 6].

Each particular constituent field in (2) gives a divergent contribution to the effective action $W[g]$. In the one loop approximation the divergent terms are local and of the zero order, linear and quadratic in curvature. In the induced gravity the constituents obey additional constraints, so that the divergences cancel each other. It is also assumed that

some of the fields have masses comparable to the Planck mass and the constraints are chosen so that the induced cosmological constant vanishes. As the result the effective action $W[g]$ is finite and in the low-energy limit has the form of the Einstein-Hilbert action

$$W[g] = -\frac{1}{16\pi G}\left(\int_{\mathcal{M}} dV\ R + 2\int_{\partial\mathcal{M}} dv\, K\right) + \dots\ , \tag{3}$$

where Newton's constant G is determined by the masses of the heavy constituents. The dots in (3) indicate possible higher curvature corrections to $W[g_{\mu\nu}]$ which are suppressed by the power factors of m_i^{-2} when the curvature is small. The vacuum Einstein equations $\delta W/\delta g^{\mu\nu} = 0$ are equivalent to the requirement that the vacuum expectation values of the total stress-energy of the constituents vanishes

$$\langle \hat{T}_{\mu\nu}\rangle = 0\,. \tag{4}$$

The value of the Einstein-Hilbert action (3) calculated on the Gibbons-Hawking instanton determines the classical free energy of the black hole, and hence gives the Bekenstein-Hawking entropy S^{BH}. Sakharov's equality (2) allows one to rewrite the free energy as the Euclidean functional integral over constituent fields on the Gibbons-Hawking instanton with periodic (for bosons) and anti-periodic (for fermions) in the Euclidean time boundary conditions. In this picture constituents are thermally excited and the Bekenstein-Hawking entropy can be expressed in terms of the statistical-mechanical entropy

$$S_R = -\mathrm{Tr}\ \hat{\rho}\ln\hat{\rho} \quad , \quad \hat{\rho} = \frac{e^{-\hat{H}/T_H}}{\mathrm{Tr}\ e^{-\hat{H}/T_H}} \quad . \tag{5}$$

Here the operator $\hat{H}$ is the canonical Hamiltonian of all the constituents.

How can an empty space (vacuum) possess thermodynamical properties? The entropy S_R arises as the result of the loss of the information about states inside the black hole horizon [7, 8]. In the induced gravity (entanglement) entropy (5) is calculated for the "heavy" constituents. This solves the problems of earlier attempts to explain the Bekenstein-Hawking entropy as the entanglement entropy of physical ("light") fields.

Consider the model of induced gravity [5, 6] that consists of a number of scalar fields with masses m_s and a number of Dirac fermions with masses m_d. Scalar fields can have non-minimal couplings and are described by actions

$$I[\phi_s, g] = -\frac{1}{2}\int (\phi_s^{;\mu}\phi_{s,\mu} + m_s^2\phi_s^2 + \xi_s R\phi_s^2)\sqrt{-g}\ d^4x \quad . \tag{6}$$

The presence of the non-minimally coupled constituents is important. In this case it is possible to satisfy the constraints on the parameters m_s, m_d and ξ_s which guarantee the cancellation of the leading ultraviolet divergencies of the induced gravitational action $W[g]$, Eq. (2). The induced Newton's constant in this model is

$$\frac{1}{G} = \frac{1}{12\pi}\left(\sum_s (1-6\xi_s)\ m_s^2\ln m_s^2 + 2\sum_d m_d^2\ln m_d^2\right) \quad . \tag{7}$$

The relation between the Bekenstein-Hawking entropy S^{BH} and the statistical-mechanical entropy S_R of the heavy constituents can be found explicitly and has the form [5]

$$S^{BH} \equiv \frac{1}{4G}\mathcal{A} = S_R - \bar{Q} \quad . \tag{8}$$

The important property of S_R is that it diverges because both fermions and bosons give positive and infinite contributions to this quantity. An additional term $\bar{Q}$ in (8) is proportional to the fluctuations of the non-minimally coupled scalar fields $\hat{\phi}_s$ on the horizon Σ and is the average value of the following operator

$$\hat{Q} = 2\pi \sum_s \xi_s \int_\Sigma \hat{\phi}_s^2 \sqrt{\gamma} d^2x \quad . \tag{9}$$

The remarkable property of the model is that for the same values of the parameters of the constituents, that guarantee the finiteness of G, the divergences of S_R are exactly cancelled by the divergences of $\bar{Q}$. So in the induced gravity the right-hand-side of (8) is finite and reproduces exactly the Bekenstein-Hawking expression.

What is the origin of the compensation mechanism in (8) and what is the statistical-mechanical meaning of the subtraction in this relation? The answer to both questions is in the properties of the operator $\hat{Q}$. The operator $\hat{Q}$ is a Noether charge [9] associated with the non-minimal couplings of the scalar fields. In statistical-mechanical computations we consider fields localized in the black hole exterior. The charge $\hat{Q}$ determines the difference between the energy $\hat{E}$ in the external region $\mathcal{B}$, defined by means of the stress-energy tensor and the canonical energy $\hat{H}$ in the same region

$$\hat{E} = \int_{\mathcal{B}} \hat{T}_{\mu\nu}\zeta^\mu d\sigma^\nu = \hat{H} - T_H \hat{Q} \, . \tag{10}$$

Here ζ^μ is the timelike Killing vector field.

Two energies, $\hat{E}$ and $\hat{H}$, play essentially different role. The canonical energy is the value of the Hamiltonian $\hat{H}$ which is the generator of translations of the system along ζ^μ and which enters definition (5) of the statistical-mechanical entropy S_R. The energy E is the contribution of the constituents to the black hole mass. The number $\nu(E)\Delta E$ of microscopically different physical states of the constituents in the energy interval ΔE near $E = 0$ determines the degeneracy of the black hole mass spectrum.

Since the Killing vector ζ^μ vanishes at the bifurcation surface Σ of the Killing horizons, the Hamiltonian $\hat{H}$ is degenerate. One can add to the system an arbitrary number of soft modes, i.e. modes with negligibly small frequencies, without changing the canonical energy. On the other hand, only soft modes contribute to the average $\bar{Q}$. According to (10), this removes the degeneracy of the energy $\hat{E}$. As the result the infinite number of thermal states of the constituents reduces to the finite number of physical states of the black hole and S_R reduces to the Bekenstein-Hawking value. The corresponding number density $\nu(E = 0)$ of physical states is $\exp S^{BH}$ [6]. There is a similarity between this mechanism and gauge theories, soft modes playing the role of the pure gauge degrees of freedom.

Let us note that the concrete model of the induced gravity may differ from the one considered here, and may contain, for example, finite or infinite number of fields of higher spins. However our consideration indicates that it is quite plausible that the same mechanism of black-hole entropy generation still works.

Our discussion can be summarized as follows. The entropy S^{BH} of a black hole in induced gravity is the logarithm of the number of different states of the constituents obeying the condition $\langle \hat{T}_{\mu\nu} \rangle = 0$. The statistical-mechanical explanation of the entropy becomes possible only when one appeals to a more deep underlying theory in which vacuum is equipped with an additional "fine" structure. The vacuum in the induced gravity is a ground state of "heavy" constituents. "Light" particles are just long wave collective excitations over this vacuum. The black hole entropy S^{BH} acquires its statistical-mechanical meaning because of this additional fine structure of the vacuum of the underlying theory. Two important elements that make statistical-mechanical picture self-consistent and universal are representation of gravity as an induced phenomenon and the ultraviolet finiteness of the induced gravitational couplings. These are main lessons the induced gravity teaches us.

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